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Energy and power fluctuations in vibrated granular gases

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Abstract. Using two-dimensional numerical simulations of a granular gas excited by vibrating one of the container boundaries, we study the fluctuations of its total kinetic energy, of the power injected into the gas by the moving boundary and of the power dissipated by inelastic collisions. We show that an effective number N_f of degrees of freedom that depends on the inelasticity of collisions can be extracted from the probability density function (PDF) of the fluctuations of the total kinetic energy E. $\langle E\rangle/N_f$ is then an intensive variable contrary to the usually defined granular temperature $T_{gr} = \langle E\rangle/N$. We then show that an intensive temperature can also be calculated from the probability of certain large deviations of the injected power. Finally, we show that the fluctuations of injected and dissipated power are related such that their ratio is inversely proportional to the square-root of the ratio of their correlation times. This allows to define a quantity homogenous to a temperature that is intensive and conserved in the process of energy dynamics from its injection by the driving piston to its dissipation by inelastic collisions.

PACS. 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 05.45.-a Nonlinear dynamics and nonlinear dynamical systems

1 Introduction

Granular matter is a canonical example of a dissipative system. Indeed in such media, when two particles collide, a part of their kinetic energy is irreversibly lost by inelastic processes. Without a continuous input of energy, the granular gas cools down and even collapses into clusters [1,2]. However, when power is injected continuously, for instance by moving a boundary of the container, a statistically stationary regime is obtained where granular matter at low enough density can behave like a gas in which all particles follow erratic motions [3]. In this state, it is tempting to use some concepts of kinetic theory to describe the granular gas. However such an extension is not obvious for dissipative systems. This is well known for high densities but we emphasize that we also expect nontrivial dissipative effects in low mean density situations. Indeed, the power is injected at the surface of the container by a moving boundary in most realistic configurations, but dissipated in the bulk by inelastic collisions. We show in Section 2 that this generates stationary states with a strongly inhomogeneous density distribution even in the absence of gravity. Accordingly, the usually defined granular temperature, i.e. the mean kinetic energy per particle, $T_{gr} = \langle E\rangle/N$, is not an intensive variable. However, we observe in Section 3 that it is possible to extract an effective number N_f of degrees of freedom from the probability density function (PDF) of the fluctuations of the total kinetic energy $E(t)$, such that $\langle E\rangle/N_f$ is intensive. We then study in Section 4 the fluctuations of the power, $I(t)$, injected by the moving piston into the granular gas, and its large deviations. Finally, we illustrate a remarkable relation between the fluctuations of the injected power and the dissipated one $D(t)$ by inelastic collisions in the bulk of the granular gas. In a stationary state, besides the obvious relation, $\langle I\rangle = \langle D\rangle$, the standard deviations $\sigma(I)$ and $\sigma(D)$ are such that $\sigma(I)^2\tau_I = \sigma(D)^2\tau_D$, where τ_X is the correlation time of X. It is therefore possible to define a quantity corresponding to a temperature, $\sigma(I)^2\tau_I/\langle I\rangle = \sigma(D)^2\tau_D/\langle D\rangle$, that characterizes the power fluctuations from injection to dissipation.

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2 Granular temperature

The simulated system is the one of reference [4]. An event driven method is used. N disks of unit mass and radius a are enclosed in a rectangular box. As in real experiments [3], the energy input is provided by one wall of size L which moves with a periodic pseudo-sinusoïdal motion approximated here with parabolas. The opposite wall is motionless, and the particle-wall collisions are elastic. Periodic boundary conditions are taken for the two other lateral walls, of size H (see Fig. 1) [5]. Here we consider only the case without gravity. The inelastic binary

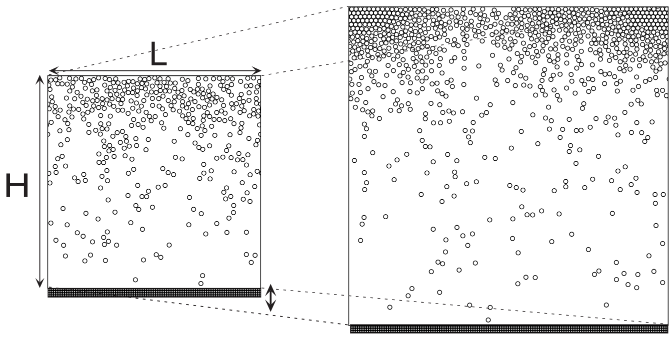

Fig. 1. Snapshot of the simulated 2D granular gas in different volumes at constant density: left cell, $N = 400$, $L = H = 100$ a, right cell, $N = 900$ $L = H = 150$.

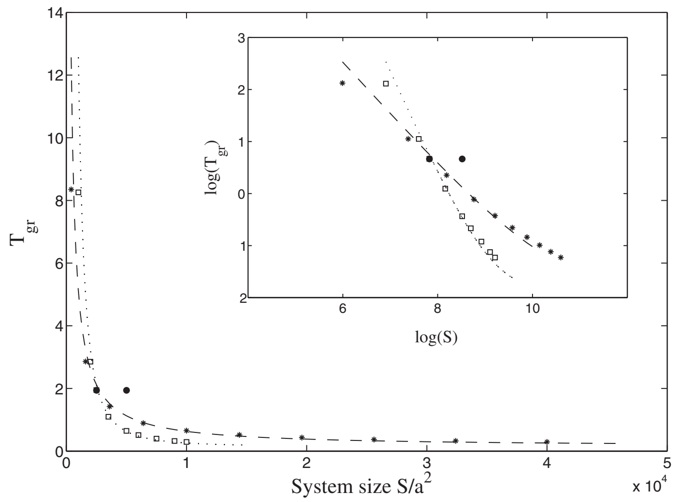

Fig. 2. The granular temperature $T_{gr} = E/N$ for different surface of the cell $S = HL$ with $\rho = N/S$ constant: (•) only L is increased, ($*$) H and L are increased, (□) only H is increased. Dashed and dotted lines represent the fits obtained when the space dependence of the density is taken into account (see text).

collisions between particles are modelled with a constant restitution coefficient, r, where r is the ratio between the pre- and post-collisional normal relative velocities. Hereafter, we take $r = 0.9$. The vibration amplitude, $A = 3a$ and the period, τ_p, taken as unit of time, are maintained constant, and we study the effect of stretching the volume of the box at constant particle density, $\rho = 0.04$.

The granular temperature is usually defined as $T_{gr} = \overline{v^2}/2$, where $\overline{v^2}$ is the mean kinetic energy per particle (and the overbar stands here for the average on time and on the particles). Depending on the way chosen to change the volume, it is generally not an intensive variable. This is clearly shown on the log-log plot of the inset in Figure 2. If L is increased proportionally to N with ρ and H constant, we just replicate the initial cell. In that case, T_{gr} is constant as long as the long wavelength instability of the homogeneous state, appearing when the aspect ratio, $\Gamma = L/H$, is large enough, does not occur [2,6]. However, if we increase H (or both L and H) with $\rho = N/(HL)$ constant, T_{gr} decreases continously. This decrease of T_{gr} with increasing H can be understood by considering both injection and dissipation phenomena.

As in most dissipative systems [9], we have an energy balance equation of the form

$$\frac{dE}{dt} = I - D. \tag{1}$$

The obvious equation $\langle I \rangle = \langle D \rangle$, obtained in the stationary regime, can be used to evaluate the granular temperature. Indeed, the mean injected power $\langle I \rangle$ can be estimated in first approximation as the 2D granular pressure p multiplied by the piston length, L, and the piston velocity, V, where, like in a non-dissipative gas, p is estimated as the kinetic energy density $\langle E \rangle / S = \rho T_{gr}$ with $S = HL$ is the area of the cell [7]. Therefore $\langle I \rangle \sim \rho T_{gr} L V$. On average, the dissipated energy at each collision is proportional to $(1 - r^2)\langle E \rangle / N = (1 - r^2) T_{gr}$. The collision frequency can be estimated by $\rho \sqrt{\overline{v^2}} a$, where $\overline{v^2} \sim T_{gr}$. Therefore the total dissipated power is $\langle D \rangle \sim (1 - r^2) N \rho a T_{gr}^{3/2}$. This gives in the stationary regime, $T_{gr} \sim V^2/((1 - r^2) a \rho H)^2$, i.e. T_{gr} does not depend on L at constant ρ but decreases when H grows. Note however that instead of a power law dependence of T_{gr} on H with an exponent -2, we get an exponent somewhere between -1.5 and -1.1. This shows the roughness of the above argument which assumes a constant piston velocity V when evaluating the injected power and does not take into account the strong inhomogeneities of the particle density which are apparent in Figure 1. This can be taken into account by defining the density $\rho(x)$ and the granular temperature $T(x)$ in a thin layer at a distance x from the piston. The pressure must be constant in the absence of gravity. The kinetic relation $p = \rho(x) T(x)$ is still valid in the dilute continuum limit. Writing the energy balance (1) in the above thin layer (I then consists of the difference between the two surface terms, i.e. the energy flux across the layer), we get $T(x)$ and observe that, although $T_{gr} = \langle T(x) \rangle$ versus H is not any more a simple power law, it displays a much better agreement with our numerical simulations (see Fig. 2). Anyway, the main point remains that the injected power does not display the same functional dependence as the dissipated one when the system size is changed at constant density. Therefore the mean total kinetic energy $\langle E \rangle$ is not an extensive variable or correspondingly T_{gr} is not intensive.

3 Energy fluctuations

In this section, we emphasize another difference with nondissipative systems at equilibrium. The temporal fluctuations of global variables such as the injected power, I, or the dissipation, D, related to their mean value, are large, and more importantly, their standard deviation, σ, does not decrease according to the $1/\sqrt{N}$ law when the

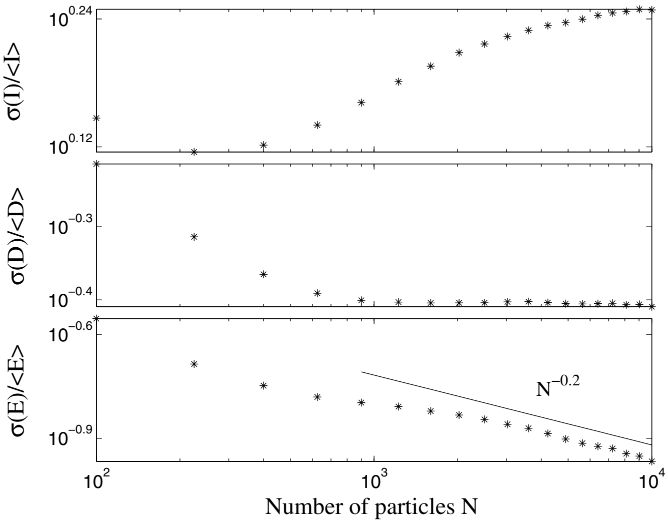

Fig. 3. Log-log representation of the relative fluctuations of the injected and the dissipated powers and the kinetic energy (from the top to the bottom respectively) versus the number of particles in a square cell. The density of particles is kept constant: $\rho = 0.04$.

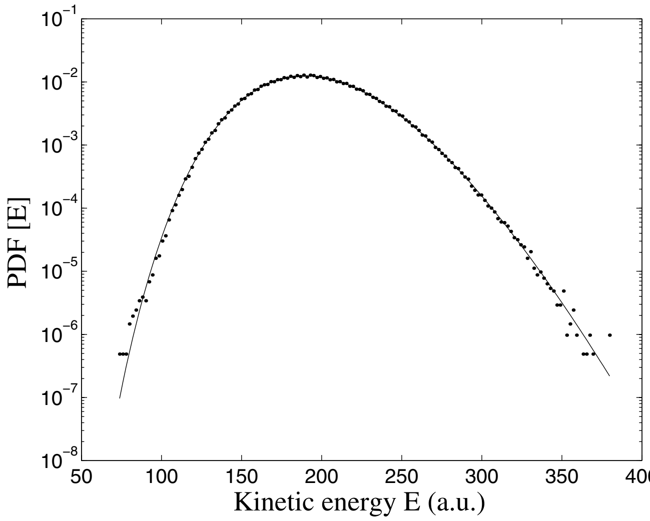

Fig. 4. PDF of the total kinetic energy, E, given by the numerical simulation with $L = H = 50$, $N = 100$ $(\cdots)$ and the best fit with a χ^2-law.

particle number is increased at constant density in a square box (see Fig. 3). Only the kinetic energy displays a power law fall-off but much more slowly than for systems at equilibrium, since $\sigma(E)/\langle E\rangle \propto N^{-\gamma}$ with $\gamma \approx 0.2$ instead of 1/2. We think that relative fluctuations which decay very slowly or even stay finite in the large N limit should be taken into account to characterize the stationary state of a granular gas using thermodynamic like concepts.

We first consider the fluctuations of kinetic energy and try to extract from them an intensive variable T_E which characterises the kinetic energy per degrees of freedom. It is known that the PDF of the fluctuations of energy of a perfect gas follows a χ^2-law. Indeed, this probability law is the one for a sum of the squares of independent random variables with a Gaussian distribution [8]. Surprisingly, as we already showed in [9], this distribution also perfectly fits energy fluctuations of a dissipative gas (see Fig. 4). Moreover, this remains true whatever the system size in the kinetic regime (i.e. when there is no clustering). The χ^2-law can be written:

$$\Pi(E) = C \cdot E^{\left(\frac{N_f}{2}-1\right)} \exp\left(-\beta_E \frac{E}{2}\right) \qquad (2)$$

where C is a normalization constant, $\beta_E = 2\langle E\rangle/\sigma(E)^2$ and $N_f = 2\langle E\rangle^2/\sigma(E)^2$ is the number of independent Gaussian variables. For a perfect gas in a two-dimensional box, $N_f = 2N$, but in the case of dissipative collisions, N_f is smaller and decreases if the restitution coefficient is decreased for N fixed [9]. N_f scales like $N^{2\gamma}$ and thus depends on the way the box is increased to keep the density constant. In a square cell, we have $N_f \propto N^{0.4}$ (see Fig. 3). When only H is increased, N_f becomes independent of N since $\sigma(E)/\langle E\rangle$ first grows with N and then saturates.

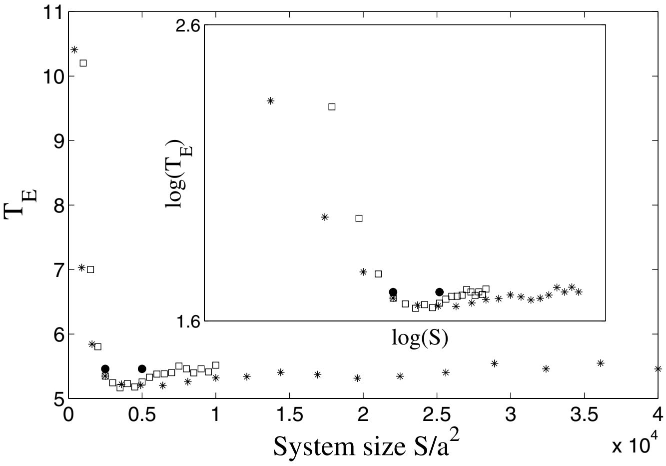

Fig. 5. The temperature $T_E = \langle E\rangle/N_f = 1/\beta_E$ for different system sizes for $\rho = N/(HL)$ constant: (•) only L is increased, $(*)$ H and L are increased, (□) only H is increased.

It is remarkable that the χ^2-distribution persists for the dissipative granular gas. This means that the total kinetic energy E is an extensive function of N_f although it is not an extensive function of N. Correspondingly, energy fluctuations decrease proportionally to $1/\sqrt{N_f}$ when N_f is increased. This leads us to propose the following definition of a granular temperature, $T_E = \langle E\rangle/N_f = 1/\beta_E$. Figure 5 indeed shows that T_E is intensive whatever means we choose to increase the system size. For large enough systems, T_E becomes constant within 5% fluctuations when the system size is increased although the density distribution in space changes. As shown in Figure 1, the system is inhomogeneous at large H, the density being larger far from the piston.

The temperature T_E is a global measurement over all the cell. It would be also interesting to try a local definition of the temperature T_E, in order to check if there is a temperature gradient in this system "heated" at one side by the piston. We do observe this tendency but there are some difficulties to measure the fluctuations of E in stripes which must be sufficiently narrow to give a local measurement but must contain enough particles to provide statistically relevant measurements.

It is difficult to give a physical interpretation of N_f: the particle velocities become more correlated when the collisions are more inelastic such that the effective number of degrees of freedom of the gas decreases. In some sense, coarse-grained velocities defined on a correlation length that increases with dissipation, behave like independent Gaussian variables and give the χ^2-distribution in first approximation. We have presently no mean to evaluate N_f from the system parameters. In addition, it would be rather difficult to measure the fluctuations of the total kinetic energy E in 3D experimental configurations because we need to know the velocities of all particles.

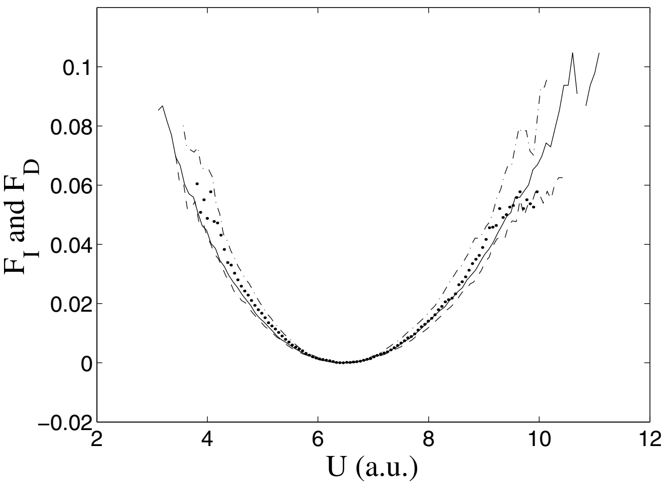

Fig. 6. The large deviation function of the injected power in a cell where $L = H = 50a$, smoothed over a time $\tau = 100\tau_p$ (—), $\tau = 150\tau_p$ (− − −) and the large deviation function of the dissipated power smoothed over a time $\tau = 100\tau_p$ (− · −), $\tau = 150\tau_p$ (· · ·).

4 Large deviations of injected power

A more easily measurable quantity is the power I, injected by the moving piston into the granular gas. More precisely, it is possible to measure a smoothed injected power

$$I_\tau(t) = \frac{1}{\tau}\int_t^{t+\tau} I(t')dt', \qquad (3)$$

over a time τ, due to the time resolution of the experimental device. Generally τ is (or can be chosen) much larger than all the "microscopic" characteristic times (collision time, particle free flight time, etc.). If τ is larger than any correlation time of the system then the large deviation theorem [10] gives the probability Π to observe a fluctuation $I_\tau(t) = U$:

$$\Pi(I_\tau(t) = U) \approx C \cdot \exp(-\tau F_I(U)), \qquad (4)$$

where F_I is the function of large deviations plotted in Figure 6. For large τ, it is a convex function with one minimum equal to zero for $U = \langle I \rangle$ [10]. Moreover, it must be an extensive function of the independent degrees of freedom of I. Actually I and then I_τ are integrated in space and for large enough systems, one expects the large deviation theorem to hold in space, i.e. we expect: $F_I(U) \sim N'_f \mathcal{F}_I(U/N'_f)$ with $\mathcal{F}_I$ intensive and N'_f the independent degrees of freedom of the system [10,12].

In order to extract a temperature from this function of large deviations, we can first follow the Fluctuation Theorem demonstrated for a class of reversible dissipative systems [13] or for Langevin type dynamics [14]. Thanks to a detailed balance in these model systems, a temperature can be obtained from the study of the probability to observe positive versus negative fluctuations of I_τ. In our system, negative values of the injected power correspond to events when a particle hits the piston during its descending motion (see Fig. 1). Since the relative number of negative events of I_τ is already small for $\tau = \tau_p$, and becomes smaller and smaller when τ is increased, we can perform a Taylor expansion of (4) around $U = 0 \pm \epsilon$:

$$\log\left(\frac{\Pi(I_\tau = +\epsilon)}{\Pi(I_\tau = -\epsilon)}\right) \approx -2\tau\left\{\epsilon\left[\frac{\partial F_I}{\partial U}\right]_{U=0}\right\}. \qquad (5)$$

The last term in the bracket is expected to be intensive (this will be checked below). It has the dimension of the inverse of an energy. The temperature deduced here within the formalism of large deviations corresponds to the one rigorously computed in the Fluctuation Theorem for a class of out-of-equilibrium reversible systems [13].

As shown in Figure 7 for a given size of the system, equation (5) is verified as soon as τ is larger than the excitation period τ_p. Since the number of negative events is small and becomes smaller and smaller when τ is increased, the Taylor expansion (5) is true for all $\tau \geq \tau_p$ but the range of experimentally observed values of ϵ becomes smaller. As conjectured in [9] and shown on a particular example in [11], the fluctuation theorem does not apply for non reversible dissipative systems but experimental or direct numerical observations follow (5) precisely because of the small range of explored ϵ.

From the slope β_F of the curve in Figure 7, we can define a temperature $T_F = 1/\beta_F$. T_F is plotted in Figure 8 for different system sizes at constant density. It becomes constant for large enough systems. It is remarkable that T_F behaves intensively, although the mean injected power $\langle I \rangle$ varies with the system size.

We should however point out a negative aspect of the definition of T_F. It strongly relies on the existence of negative events of the injected power and thus confers a specific role to the null injected power. This is fine for reversible dissipative systems for which the Fluctuation Theorem has been demonstrated [13]. However, many realistic dissipative systems do not involve negative injected power or

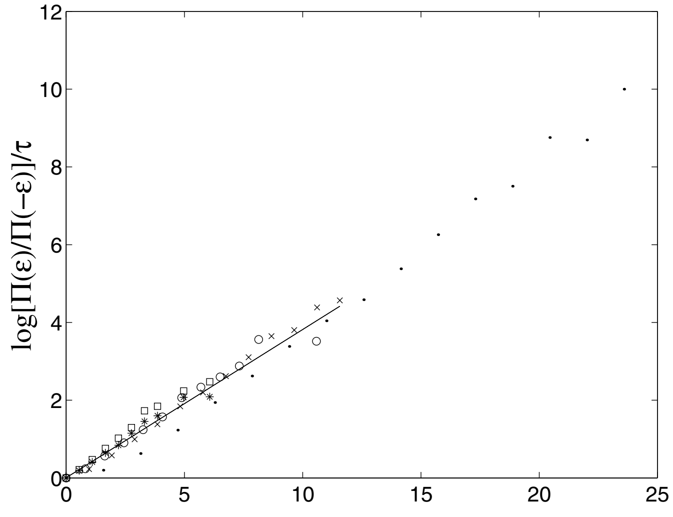

Fig. 7. Logarithm of the probability to observe a fluctuation $I_\tau = \epsilon$ divided by the probability to observe a fluctuation $I_\tau = -\epsilon$ for different values of τ: $(\cdots)$ $\tau = \tau_p$, $(\times\times\times)$ $\tau = 2\tau_p$, (ooo) $\tau = 3\tau_p$, (□□□) $\tau = 4\tau_p$ and $(***)$ $\tau = 5\tau_p$. $L = H = 50a$.

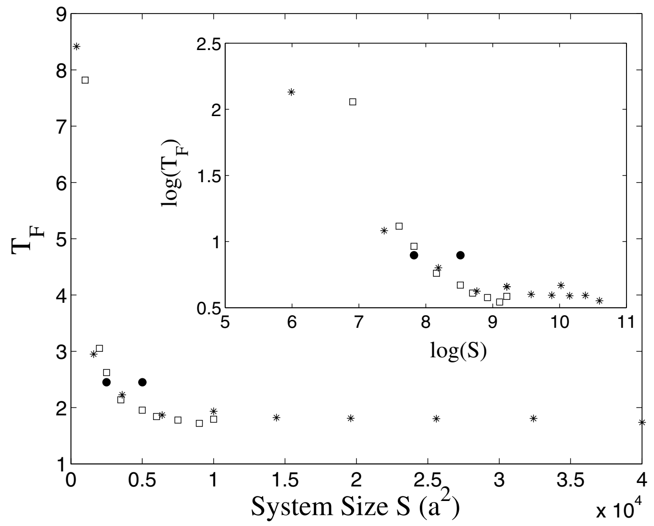

Fig. 8. The temperature $T_F = 1/\beta_F$ for different system sizes for $\rho = N/(HL)$ constant: (•) only L is increased, $(*)$ H and L is increased, (□) only H is increased.

their existence strongly depend on the particular way the power is injected. For a granular gas excited by a moving piston, negative events are almost never observed if the motion of the piston is an asymmetrical ramp with very short descending motions. In addition, even for an excitation which provides negative events for the injected power, the dissipated power D is always positive. Therefore, it is not possible to use the same procedure to extract a temperature characterizing the fluctuations of the dissipated power. Therefore, another approach will be considered in the next section.

5 Characteristic fluctuations of the injected or dissipated power

As said above, an extrapolation of the ideas stemming from the Fluctuation Theorem to most experimental dissipative systems displays some difficulties, especially due to the possible lack of negative events in the injected power. Moreover, the fluctuations of the injected and dissipated power cannot be characterized in the same way.

In order to define an observable playing the role of a "temperature", that is, being intensive, measurable, and efficient in characterizing more or less the level of fluctuations, we consider more closely the properties of the conservation equation (1). In the stationary regime, one deduces the obvious equation $\langle I\rangle = \langle D\rangle$. It is however possible to use that equation once more to derive other types of balance equations. For instance, let us define for an observable X

$$C_X \equiv \lim_{t\to\infty} \int_0^t \left[\langle X(t')X(0)\rangle - \langle X\rangle^2\right] dt'. \qquad (6)$$

Using equation (1) to eliminate I in the expression of C_I and using time translation invariance implied by stationarity to evaluate the integral on t' together with the relation $\langle I\rangle = \langle D\rangle$ leads to the important equality [15]

$$C_I = C_D. \qquad (7)$$

Thus, a characteristic energy $T_I = C_I/\langle I\rangle$ can be defined from the statistical properties of the injection mechanism — accessible to experiments — and displays a very interesting property: the corresponding quantity $T_D = C_D/\langle D\rangle$ associated with the dissipation is *strictly* the same. As a result, the temperature defined in such a way is intrinsically related to the global process of energy dynamics throughout the system, since at both ends of the energy transfer mechanism, its value is the same.

There is another way of understanding relation (7) if one assumes the form $\sigma^2(X)\, g_X(t/\tau_X)$ for the time-displaced correlations of X, where $g_X(t/\tau_X)$ is a dimensionless function that decays in an integrable way at large times. Then, (7) can be written in the form

$$\frac{\sigma(I)}{\sigma(D)} = \sqrt{\frac{\tau_D}{\tau_I}}, \qquad (8)$$

where τ_X is the typical correlation time of $X = I$ or D and $\sigma(X)$ the standard deviation. This shows that the relative fluctuations are larger when their correlation time is smaller. This has been observed in other dissipative systems, for instance a shell model of turbulence for which dissipation fluctuates much more rapidly than power injection but at the same time, displays a much larger relative amplitude [9].

In addition, C_X where $X = I$ or D is related to the curvature of the large deviation function around $\langle X\rangle$. Indeed, due to the large deviation theorem, the probability to get a fluctuation

$$\Delta X_\tau = 1/\tau \int_t^{t+\tau} X(t')dt' - \langle X\rangle$$

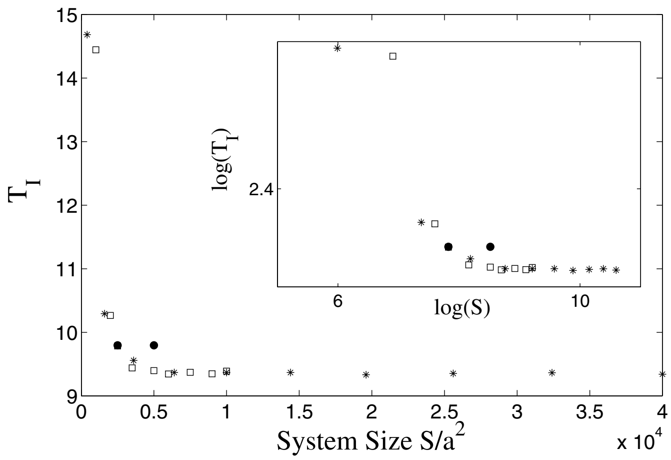

Fig. 9. The temperature $T_I = 1/\langle I \rangle \left[\partial^2 F/\partial U^2\right]_{U=\langle I \rangle}$ for different system sizes for $\rho = N/(HL)$ constant: (•) only L is increased, (∗) H and L are increased, (□) only H is increased.

at large τ is essentially given by:

$$\Pi(\Delta X_\tau) \approx C \cdot \exp\left(-\frac{\tau F''_{\langle X \rangle}}{2} \Delta X_\tau^2\right), \tag{9}$$

where $F''_{\langle X \rangle} = \left[d^2 F_X / dX^2\right]_{X=\langle X \rangle}$. Therefore

$$\sigma(X_\tau)^2 \approx \frac{1}{\tau F''_{\langle X \rangle}}. \tag{10}$$

A suitable change of the integration variables and stationarity also impose

$$\sigma(X_\tau)^2 \equiv \frac{1}{\tau^2} \left\langle \int_t^{t+\tau} \Delta X(t_1) dt_1 \int_t^{t+\tau} \Delta X(t_2) dt_2 \right\rangle \tag{11}$$

$$= \frac{2}{\tau^2} \int_0^\tau (\tau - t) \langle \Delta X(t) \Delta X(0) \rangle \, dt. \tag{12}$$

When τ tends to infinity, (6) and (11) give at the first order in $1/\tau$: $C_X \approx 1/(2F''_{\langle X \rangle})$ and we have from (7), $F''_{\langle I \rangle} \approx F''_{\langle D \rangle}$. This is shown in Figure 6 where the curvature of F_I and F_D have become equal for large τ.

For our granular gas, T_I is displayed in Figure 9 and behaves intensively when the system is large enough.

As for T_E, such a concept is liable to be generalized to local versions of this temperature, since a part of the system can always be thought as a dissipative open subsystem exchanging energy with its neighbourhood. However, the fluctuations of the energy flux coming into it should be dominated by a particle flux term. Indeed, if we estimate the relative fluctuations of the number of particles, $\sigma(N)/\langle N \rangle$, in a subsystem containing the third of the square cell ($50a \times 50a$) near the piston, then $\sigma(N)/\langle N \rangle$ can go up to 20% (see Fig. 1). This changes the physical source of the fluctuations. Anyway, as for T_E, such local measurements could give interesting results but are not the purpose of this work.

6 Conclusion

To conclude we would like to emphasize that the study of the fluctuations of global variables like the total kinetic energy — fluctuations which are far from being negligible in out-of-equilibrium systems — takes into account the correlations induced by the dissipation and gives a direct access to some effective numbers of degrees of freedom. It is then possible to define a "temperature" which is intensive in contrast to the usual granular temperature.

The study of this number of degrees of freedom versus the number of particles should be helpful to understand some phenomena in granular matter which look like *phase transitions*. For instance, we plan to use the concepts introduced here to study the long wave instability that occurs when the piston size is increased, with H and ρ constant. This clustering transition must be of a nature very different from the stratification shown Figure 1 which is essentially due to the fact that injection is less and less efficient compared to dissipation when H is increased.

The fact that the kinetic energy follows a χ^2-law — even in the largest boxes where the density is inhomogeneous (see Fig. 1) — could be surprising. However, it must be noted that the kinetic energy and its fluctuations are essentially due to the less dense part. For instance, in the cell where $Lx = 50a$ and $Ly = 140a$ (with $\rho = 0.04$ and $r = 0.9$), the bottom third of the cell, i.e. near the piston, contains on average only 4% of the particles but 35% of the kinetic energy, whereas the top third contains 84% of the particles but contributes only to 25% of the kinetic energy. In that sense, the system is always in a dilute regime. In fact, a local estimation of the number of effective degrees of freedom gives almost the same number of *effective independent particles* in the top third as in the bottom third of the container. This underlines the higher correlations between particles in the more dense part where there are more dissipative collisions per unit of time.

The relation (8) between the fluctuations of the injected I and dissipated D power could also look surprising. Indeed, for a stationary regime of a turbulent flow for instance, it is believed that dissipative scales display some universal features whereas the dynamics of the injected power strongly depends on the way the fluid is driven. In turbulence as well as for granular flows, it is experimentally possible to decrease the fluctuations $\sigma(I)$ using a feed-back loop in order to try to maintain a nearly constant injected power, whereas it is not possible to have direct control on $\sigma(D)$ and τ_D that depend on the small scale or microscopic dynamics. (8) gives a constraint on the fluctuations of the dissipated power once the properties of the injected power are known. It would be interesting to see how $\sigma(D)$ and τ_D change when $\sigma(I)$ and $\tau(I)$ are modified by the driving. Finally, relation (8) allows definition of a "temperature" that characterizes the fluctuations of both the injected and dissipated powers.

At this stage of our study, it is clear that a lot remains to be understood about this new approach: what is for example the very physical interpretation of these "temperatures", beyond a fluctuation measurement of the energy injection-dissipation mechanism? Moreover, what is their connection with the underlying full dynamics? Could they furnish us with a valuable tool of disorder probing, or are they somehow connected to the direction of the transmission of heat?

Moreover, it would be convincing to connect our nonequilibrium "temperatures" with the Fluctuation-Dissipation Theorem in the quasi-equilibrium state as it is done for the Fluctuation Theorem [16]. However, notice that in our specific system, the non-dissipative case is a singular limit. Indeed, as long as the dissipation is larger than zero, i.e. as long as r is less than 1, there is a stationary state with $\langle I \rangle = \langle D \rangle$. For instance, $\langle I \rangle$ is 10 times larger for $r = 0.99$ than for $r = 0.9$ (with the same excitation), but a high energy stationary state is still reached where $N_f \approx 2N$. But as soon as $r = 1$, the stationary regime disappears, i.e. $\langle E \rangle$ and σ_I increase with time whatever the excitation intensity.

Finally, to motivate experimental measurements of the "temperatures" defined above, it is interesting to know that in a more realistic simulation, where the value of the restitution coefficient r depends on the relative velocity of the two colliding particles, the correlations are strongly increased (the number of effective degrees of freedom, N_f, extracted from the fluctuations of E, can be reduced by an order of magnitude). Therefore, we hope that all the effects presented must be easily measurable in a real experiment, even in a 3D system with more particles than in our simulations.

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